Intro. to ODEs

Quiz 1 Name: ____________________

1) Compute the derivatives of the following functions.

a) $g(x) = \dfrac{e^x}{1+e^{2x}}$

$$g'(x) = \frac{e^x(1+e^{2x}) - e^x(2e^{2x})}{(1+e^{2x})^2} = \frac{e^x - e^{3x}}{(1+e^{2x})^2}$$

b) $h(x) = \ln\left(\dfrac{x^2+1}{x^4+1}\right)$

$$h'(x) = \frac{x^4+1}{x^2+1}\left(\frac{2x(x^4+1)-(x^2+1)(4x^3)}{(x^4+1)^2}\right) = -\frac{2x(x^4+2x^2-1)}{(x^4+1)(x^2+1)}$$

2) Compute the following indefinite integrals.

a) $\displaystyle\int xe^{-x}\,dx$

Use integration by parts with $u = x$ and $dv = e^{-x}dx$.

$$\begin{aligned}\int xe^{-x}\,dx &= -xe^{-x} + \int e^{-x}\,dx \\ &= -xe^{-x} - e^{-x} + C\end{aligned}$$

b) $\displaystyle\int \ln(x)\,dx$

Use integration by parts with $u = \ln(x)$ and $dv = dx$.

$$\begin{aligned}\int \ln(x)\,dx &= x\ln(x) - \int 1\,dx \\ &= x\ln(x) - x + C\end{aligned}$$

c) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

Use the substitution $u = e^x$.

$$\begin{aligned}\int \frac{e^x}{1+e^{2x}}\,dx &= \int \frac{1}{1+(e^x)^2}\,e^x dx \\ &= \int \frac{1}{1+u^2}\,du = \arctan(u) + C \\ &= \arctan(e^x) + C\end{aligned}$$

3) Compute the following partial derivatives of

$$g(x,y) = x^2\cos(4x^3 - 2y^2).$$

a) $\dfrac{\partial g}{\partial x}(x,y)$

$$\frac{\partial g}{\partial x}(x,y) = 2x\cos(4x^3 - 2y^2) - 12x^4\sin(4x^3 - 2y^2)$$

b) $g_{yy}(x,y)$

$$\begin{aligned} g_y(x,y) &= 4x^2 y\sin(4x^3 - 2y^2) \\ g_{yy}(x,y) &= 4x^2\sin(4x^3 - 2y^2) - 16x^2y^2\cos(4x^3 - 2y^2) \end{aligned}$$

4) Consider the complex number $z = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$.

a) Write z in polar form $z = re^{i\theta}$.

z is in Quadrant I of the complex plane at an angle of $\pi/4$ from the positive real axis. Since

$$r = |z| = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1,$$

we must have

$$z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = e^{i\pi/4}.$$

b) Use the polar form to compute z^{16}.

$$\begin{aligned} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^{16} &= \left(e^{i\pi/4}\right)^{16} \\ &= e^{4\pi i} \\ &= \cos(4\pi) + i\sin(4\pi) \\ &= 1 \end{aligned}$$